MATH2060B: Analysis II: Home Project 1 **26 Feb 2021 at 5:00pm**

Important Notice:

♣ The answer paper **Must be submitted before 27 Feb 2021 at 5:00pm.**
♠ The answer paper MUST BE sent to the CU Blackboard.
✠ The answer paper **Must include your name and student ID.**

Answer ALL Questions

1. **(10 points)**
   Let $f(x) = sgn(\sin\frac{\pi}{x})$ for $x \neq 0$ and $f(0) = 0$, where sgn denotes the sign function. Show that f is Riemann integrable over $[-1, 1]$ and find $\int_{-1}^{1} f(x)dx$.

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2. **(20 points)**

Let f be a continuous real-valued function defined on $\mathbb{R}$.

(a) Suppose that there are constants c_0 and c_1 such that

$$\lim_{x \to 0} \frac{f(x) - c_0 - c_1 x}{x} = 0.$$

Show that $f'(0)$ exists.

(b) Suppose that f is a C^1-function and there are constants c_0, c_1 and c_2 such that

$$\lim_{x \to 0} \frac{f(x) - c_0 - c_1 x - c_2 x^2}{x^2} = 0.$$

Does it imply that the second derivative of f at 0 exist?

3. **(20 points)**

Let $f : (0,1) \to \mathbb{R}$ be a function given by

$$f(x) = \begin{cases} \frac{1}{p} & \text{if } \ x = \frac{q}{p} \text{ and } p, q \text{ are relatively prime positive integers;} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

(a) Describe the continuity of f.

(b) Describe the differentiability of f.
Justify your answer by using the definitions.

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